Energy recovery in an optical linear collider

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It is demonstrated that recovery of the electromagnetic energy of the fundamental mode at the output of an acceleration structure leads to a significant efficiency enhancement. When using a single bunch, the number of electrons accelerated is rather small. In fact, this number is virtually identical to the case when no feedback loop is employed. To increase this number, in parallel with the efficiency enhancement associated with the feedback loop, it is necessary to split the bunch into a train of microbunches—this last process leads to suppression of high-order modes.

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I. INTRODUCTION

To a large extent, the fate of any future electron accelerator will be determined by its efficiency. Typically this is only a few percent; therefore, the natural question is whether it is possible to significantly increase the overall efficiency of the system. In principle, the answer is positive since concepts similar to those employed in microwave vacuum tubes may be applied to accelerators. To envision an existing implementation consider a beam of electrons generated on a cathode to which a negative potential is applied, say −400 kV. If in the (grounded) interaction region, 20% of the kinetic energy is converted into microwave radiation, then, in principle, applying a negative potential of −320 kV on the collector, may lead to an overall efficiency approaching the 100% level. This is the so-called *depressed collector* method and it was applied in the past to traveling wave tubes and klystrons and more recently to free electron lasers and gyrotrons [1].

Conceptually, a similar method may be applied to "short" accelerators although its implementation is quite different, since clearly the decelerating potentials are not available. In 1965 Tigner [2] suggested decelerating the accelerated bunch by reinjecting it into a periodic structure, thus forcing it to generate radiation and use the latter in order to accelerate a different bunch of electrons. In other words, energy is *indirectly* "recycled" after being recovered from electrons that are eventually dumped. A recent experiment at Jefferson Labs demonstrated an energy recovery efficiency of 75% [3,4], making the energy recovery linac (ERL) a leading candidate for a new generation of high-brilliance x-ray sources [3–6].

Three main ERL configurations for x-ray generation are being currently considered in the United States: one is a collaboration of Cornell University and Jefferson Labs [7], a second one is at BNL [8], and a third is at LBL [9]. In the long run the Cornell ERL is planned to deliver a 7 GeV beam with 100 mA of current but as a proof of principle a down-scaled system 100 MeV–100 mA is being explored at this stage. The proposed BNL system aims at less than half the energy (3 GeV) but somewhat higher current (100–200 mA), whereas the LBL intends to focus on very short pulses (femtosecond scale) at energies comparable to the BNL machine (2.4 GeV). Each one of these sets of parameters emphasizes the importance of the energy recovery concept. Taking as a figure of merit the Cornell group long term plan, the 7 GeV–100 mA beam entails 700 MW of power which without an energy recovery scheme is a prohibitively high average power.

Another aspect that has a profound impact on the concept presented here is the advantage associated with recent progress in solid-state technology, indicating that this may reach wall plug to light efficiencies of 30% in the relatively near future. In order to envision the advantage of operating in the optical regime, it is sufficient to extrapolate the scaling law developed for microwave machines to the optical regime: the gradient is proportional to the square root of the power and inversely proportionally to the wavelength $G \propto \sqrt{P}/\lambda$. Consequently, reducing the operating wavelength by five orders of magnitude (from 10 cm to 1 μm) implies a reduction of power of ten orders of magnitude provided the gradient is kept the same. As an example, let us consider qualitatively the next linear collider (NLC) design: the anticipated gradient is $G \simeq 100$ MV/m, the power injected (after compression) in a single acceleration module is of the order of $P \simeq 250$ MW, and roughly the operating wavelength is $\lambda \simeq 3$ cm; these parameters correspond to an interaction impedance $Z_{\text{int}} \simeq (G\lambda)^2/P \sim 36$ kΩ. If for the sake of the present discussion we assume that Z_{int} remains the same when the system is scaled to operate at 1 μm with a gradient of 1 GV/m then the necessary power is less than 30 W. In practice, as we shall discuss in more detail in what follows, the interaction impedance of an optical structure is three orders of magnitude smaller and, as a result, the peak power is by three orders of magnitude higher. Nevertheless, several kilowatts of laser power are definitely within reach, making the optical schemes particularly appealing. Although we hinted at the resemblance between a microwave and an optical acceleration structure, there is a profound difference between the two as reflected in this study: at optical wavelengths, dielectrics sustain higher electric fields and therefore future acceleration structures are anticipated to be made of dielectrics.

It is our goal in this publication to demonstrate that the recycling concept is not limited only to "small" machines but may be implemented in a full optical collider by *directly* recovering the electromagnetic energy within each accelera-

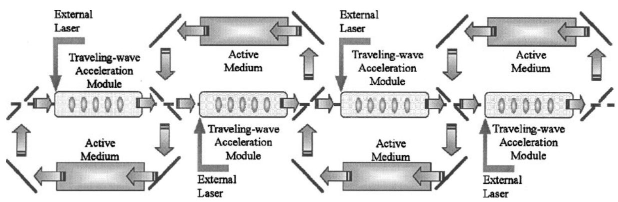

FIG. 1. Schematic configuration of a possible implementation of direct electromagnetic energy recycling in an optical accelerator. A macrobunch consisting of a few hundreds of microbunches generates a wake that is virtually coherent with the accelerating laser field. The superposition of accelerating field and wake is amplified in the active medium, providing the energy increment required to maintain stable equilibrium.

tion module and obviously without dumping the electrons. What makes this concept feasible is the fact that, rather than accelerating a single bunch, in an optical linear collider one *macrobunch* consisting of many hundreds of microbunches is being accelerated—each microbunch being one laser wavelength (λ) apart from its neighbor. With a large number of microbunches, the *wake* generated by the macrobunch is virtually coherent with the laser field that accelerates the microbunches. Consequently, at the output of a given traveling wave acceleration module, the two field components (laser and wake) are practically inseparable and the total electromagnetic field may be extracted from the acceleration structure, amplified by an active medium, and reinjected into the traveling wave acceleration module. Figure 1 illustrates the paradigm schematically.

In order to evaluate the significance of this paradigm, let us examine some figures of merit. Typical collider designs (500 GeV) predict 8 MW of average beam power assuming 10^{14} electrons/sec. At a laser-acceleration efficiency of 8%, as preliminary estimates indicate, the acceleration structure will require about 0.1 GW of average laser power and assuming a wall plug to light efficiency of 30%, the total amount of average power necessary is about 0.34 GW. Increasing the efficiency from 8% to 40% entails a reduction in the *average power* from 340 MW to less than 70 MW. According to preliminary estimates to be presented next, such an increase, and more, is within reach. From the accelerating gradient perspective we can look at two other systems: the present Stanford linear collider (SLC) operates at roughly 20 MV/m gradient, while the next linear collider is proposed to operate at 100 MV/m; thus for a 1 TeV final energy its length should be about 10 km, whereas an optical collider may reach this energy within 1 km since the typical accelerating gradient anticipated is of the order of 1 GV/m.

As a first step in the current approach we examine the efficiency when accelerating a single microbunch in a structure without feedback; the second step is to repeat this procedure in the case of multiple microbunches; and the third is to reexamine the efficiency of accelerating a single microbunch in the presence of a feedback loop. In the last stage, multiple microbunches and a feedback system are combined for evaluating the efficiency of acceleration of the electrons.

Throughout this study we shall employ an *idealized model* in order to convey the essentials of the concept, including, in particular, *dielectric* acceleration structures that have been developed in recent years, e.g., photonic band gap [10] or Bragg [11] structures. As already mentioned, this choice of structure is dictated by inherent properties of materials since in the optical regime dielectric materials sustain much higher electromagnetic fields than do metals. Moreover, all the bunches are assumed to be pointlike, ignoring both their geometric size as well as the finite momentum spread in the transverse and longitudinal directions.

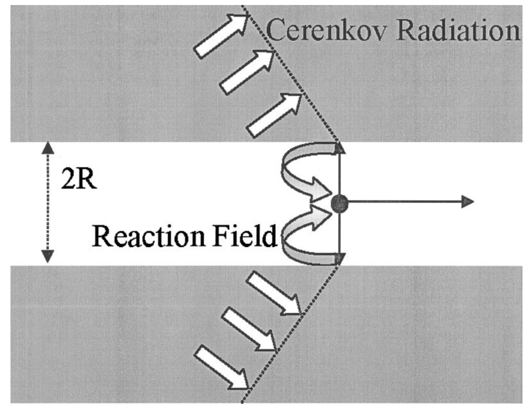

FIG. 2. A point charge moving in a vacuum tunnel bored in a dielectric material generates Čerenkov radiation. The emitted power may be determined by using the reaction field, which decelerates the particle.

II. SINGLE BUNCH AND NO FEEDBACK

In order to have a rough estimate [12] of what may be the efficiency of an optical accelerator without a feedback system when accelerating a single microbunch, let us assume that the average laser power injected in the structure is P_L and the resulting gradient at the location of the electrons is G_0. By virtue of the linearity of the Maxwell equations, these two quantities are related, defining the so-called interaction impedance $Z_{\text{int}} \equiv |G_0\lambda|^2/P_L$ characterizing any acceleration structure operating at a frequency corresponding to a vacuum wavelength λ. This laser pulse accelerates a *point charge* (q) so that as it moves in an arbitrary acceleration structure it generates an electromagnetic wake. Associated with this wake there is a decelerating electric field (E_{dec}) which again, by virtue of the linearity of Maxwell's equations, must be proportional to the charge, namely, $E_{\text{dec}}=\kappa q$ where the value of κ depends on the details of the structure; it will be referred to as the *wake coefficient*. By virtue of the fundamental loading theorem, the amplitude of the trailing wake is twice this value, namely, $E_W=2E_{\text{dec}}=2\kappa q$.

In the case of a uniform dielectric medium filling the entire space except a vacuum tunnel (see Fig. 2) of radius R, along which the point charge propagates, the wake coefficient is

$$\kappa = \frac{1}{4\pi\varepsilon_0 R^2} \times 2. \tag{2.1}$$

This result has been demonstrated analytically in Ref. [13] and it can be demonstrated for a metallic wall waveguide loaded with a dielectric layer and a vacuum tunnel in the center (partially loaded dielectric waveguide). It is valid for a Bragg concentric fiber [11,14] and it was concluded to be true for any azimuthally symmetric dielectric structure having its vacuum-dielectric discontinuity at R. Although these structures are very different, from the perspective of an ultrarelativistic point charge propagating virtually at c, what counts is only the vacuum-dielectric discontinuity since this is the only discontinuity that generates a reflected wave that may affect the point charge. Any reflection occurring further away from the first discontinuity reaches the structure's axis only after the point charge has passed—thus it may affect only trailing microbunches as will be discussed subsequently.

Consequently, the spatial behavior of the *total* electromagnetic power generated by such a point charge is $P \simeq cqE_{\rm dec}$ independent of the transverse variations beyond the vacuum-dielectric discontinuity. Obviously, the behavior of the trailing field on axis is strongly dependent on the transverse characteristics of the structure. For example, in the case of a uniform structure, like the one illustrated in Fig. 2, the spectrum is continuous [12], and the wake decays exponentially in time (t) and in space (z):

$$E_z(r=0,\tau\equiv t-z/c)=q\kappa[2e^{-\tau/\tau_0}h(\tau)], \tag{2.2}$$

wherein $\tau_0^{-1}\equiv c\varepsilon/R\sqrt{\varepsilon-1}$. On the other hand, in the case of a partially loaded dielectric waveguide the spectrum is discrete [15]:

$$E_z(r=0,\tau\equiv t-z/c)=q\kappa\sum_{n=1}^{\infty}W_n\cos(\omega_n\tau)[2h(\tau)] \tag{2.3}$$

and the wake has an oscillatory character. W_n are weighting functions that may be determined analytically and can be shown to satisfy $\Sigma_{n=1}^{\infty}W_n=1$ with ω_n representing the discrete spectrum of frequencies generated by this point charge; $h(u)$ is the step function

$$h(u)=\begin{cases}0, & u<0,\\ 0.5, & u=0,\\ 1, & u>0.\end{cases}$$

For example, in the idealized case of a dielectric loaded waveguide of radius $R_{\rm ext}$ and a bunch of radius R_b, disregarding scattering processes, the relative weight of each mode is given by

$$W_n\equiv\left[\frac{2J_1(p_nR_b/R_{\rm ext})}{p_nJ_1(p_n)}\right]^2; \tag{2.4}$$

p_n are the zeros of the Bessel function of order zero and the first kind $[J_0(p_n)\equiv 0]$. Obviously, the first mode in this representation is the one designated to accelerate the electrons having a phase velocity c, group velocity $c\beta_{\rm gr}$, and interaction impedance $Z_{\rm int}$. These quantities are related to the wake coefficient (κ) and, in particular, it is possible to establish [16] the "projection" of the total deceleration on the fundamental mode (superscript F) represented by

$$\kappa^{(F)}\equiv\kappa W_1=\frac{\beta_{\rm gr}}{1-\beta_{\rm gr}}\frac{Z_{\rm int}}{\sqrt{\mu_0/\varepsilon_0}}\frac{\pi}{4\pi\varepsilon_0\lambda^2}. \tag{2.5}$$

In other words, this is the coefficient that, given the charge of the bunch, determines the amplitude of the fundamental mode generated by the bunch ($E^{(F)}=2\kappa^{(F)}q$).

In the absence of an accelerating gradient G_0, the electron bunch is decelerated along a distance d in the structure and the loss of kinetic energy is $\Delta U_{\rm kin}=-q^2\kappa d$. Consequently, when the gradient is not zero, the net change in the kinetic energy of the bunch traversing the same structure is given by $\Delta U_{\rm kin}=q(G_0-q\kappa)d$. As reference, the total electromagnetic energy stored in the structure is $U_{\rm EM}=P_L\tau_{\rm EM}$, wherein

$$\tau_{\rm EM}\equiv\frac{d}{c}\left(\frac{1}{\beta_{\rm gr}}-1\right); \tag{2.6}$$

see Appendix A. This last expression takes into consideration the requirement that the point charge and the electromagnetic pulse ought to *overlap* during the time the latter spends in the structure. Moreover, in the same Appendix it is illustrated that it equals the delay between the front end of the laser pulse and the point charge:

$$\tau_D\equiv\frac{d}{c}\left(\frac{1}{\beta_{\rm gr}}-1\right) \tag{2.7}$$

resulting from the different propagation velocities $c\beta_{\rm gr}$ and c, respectively. With these two energy definitions, the efficiency of the acceleration process may be determined by

$$\eta\equiv\frac{\Delta U_{\rm kin}}{U_{\rm EM}}=\eta_{\max}\frac{4q(q_0-q)}{q_0^2}, \tag{2.8}$$

wherein $q_0=G_0/\kappa$ is the charge for which the effective gradient vanishes, and the maximum value of the efficiency is

$$\eta_{\max}\equiv\frac{\kappa^{(F)}}{\kappa}=W_1 \tag{2.9}$$

occurring for $q=q_{\rm opt}\equiv q_0/2$. As a typical example we consider the parameters of a photonic band-gap structure as calculated in Ref. [10]: $Z_{\rm int}\simeq 20\ \Omega$, $\beta_{\rm gr}\simeq 0.6$, and $R\simeq 0.7\ \lambda$; according to these values the *maximum efficiency is* 6%. Assuming that the threshold for breakdown limits the power to 7 kW, the optimal number of electrons to be accelerated to reach this efficiency is 6×10^4. Evidently, considerations of energy spread will lead to charges and efficiencies below these optimum values.

Although this result relies on an *idealized* model, it emphasizes the motivation behind the present study: in the absence of a feedback loop, more than 90% of the electromagnetic energy is wasted; therefore, by "recycling" part of this energy we may significantly improve the efficiency of an optical system. Moreover, the analytic result in Eq. (2.9) reflects the fact that the efficiency is determined by the relative projection of the wake on the fundamental mode (W_1). This quantity may be enhanced by splitting the bunch into a train of microbunches each one separated by the wavelength of the fundamental. One last comment in this context before we proceed and determine the efficiency when the bunch is split

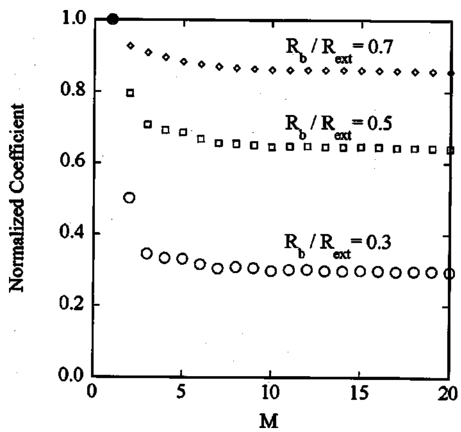

FIG. 3. Normalized wake coefficient ($\overline{\kappa}$) as a function of the number of microbunches (M).

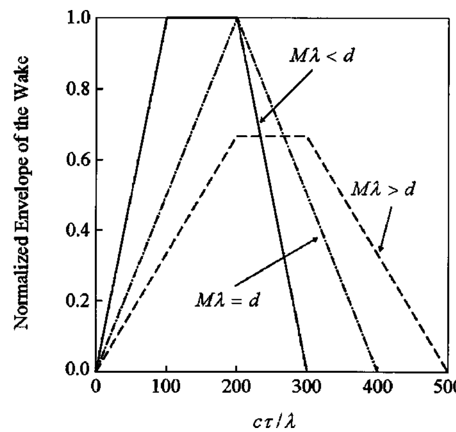

FIG. 4. The normalized envelope of the wake for the three possible cases, namely, the pulse length is longer than, shorter than, or equal to that of the structure. If $M\lambda = d$ the pulse shape is triangular; otherwise, it is trapezoidal.

into a train of microbunches: since throughout this study we shall refer to the ratio of the projection of the wake emitted power onto the fundamental represented by $\kappa^{(F)}$ and the wake coefficient (κ), we define the quantity

$$\kappa_r \equiv \frac{\kappa^{(F)}}{\kappa} \tag{2.10}$$

as the *relative wake coefficient*.

III. MULTIPLE MICROBUNCHES AND NO FEEDBACK

In order to illustrate the effect of splitting the macrobunch into a train of microbunches, we examine first the power generated by such a macrobunch and compare it to the power generated by a single point charge—both carrying the same amount of charge. According to Eq. (2.3) the latter is given by $P = qcE_{\rm dec} = q^2 c\kappa$, whereas splitting the point charge into M pointlike microbunches entails

$$P(M) = q^2 c\kappa \sum_{n=1}^{\infty} W_n \frac{\mathrm{sinc}^2[\pi M\omega_n/\omega_1]}{\mathrm{sinc}^2[\pi\omega_n/\omega_1]} \equiv q^2 c\kappa\overline{\kappa}, \tag{3.1}$$

wherein $\mathrm{sinc}(x) \equiv \sin(x)/x$; see Appendix B.

Two conclusions may be immediately drawn from this expression: first, the projection of the emitted power on the fundamental ($n=1$) is *independent* of the number of microbunches (M) or, explicitly, $\kappa^{(F)}$ does not vary as a function of the number of microbunches. Second, the sinc function acts as a filter that suppresses the contribution of virtually all the frequencies higher than the fundamental ($n > 1$). Consequently, as expected, the relative weight of the fundamental mode increases and the relative wake coefficient κ_r approaches unity, leading, according to Eq. (2.9), to higher efficiency.

For the case of a dielectric loaded structure, the normalized wake coefficient ($\overline{\kappa}$) as defined in Eq. (3.1) is illustrated in Fig. 3 with the bunch radius (R_b) as a parameter normalized to the radius ($R_{\rm ext}$) of the waveguide. Evidently it decreases rapidly from unity to an asymptotic value that depends on the bunch size and on the structure's characteristics. Bearing in mind that M affects only the nonfundamental modes, then all the reduction in $\overline{\kappa}$ is reflected in the ratio $\kappa^{(F)}/\kappa$, which qualitatively was shown above to represent the maximum efficiency developed in Eq. (2.9). Since $\overline{\kappa}$ was shown (Fig. 3) to drop by a factor of almost 5 correspondingly, the efficiency may be expected to increase by a factor of 5.

While Eq. (2.9) which has been developed for a *single* bunch reveals a general trend, it does not reflect adequately the nature of the interaction between a train of microbunches and the waves in an acceleration structure. There are two major differences that ought to be accounted for. First, for *full overlap* of the macrobunch with the electromagnetic pulse, the latter's duration must be

$$\tau_{\rm EM} = \frac{d}{V_{\rm gr}} + \frac{(M-1)\lambda - d}{c}. \tag{3.2}$$

Second, contrary to the previous section, the accelerating gradient is nonuniform since it must compensate for the beam loading effect in order to ensure *uniform acceleration* of all microbunches. This loading is qualitatively illustrated in Fig. 4 which shows the *envelope* of the projection of the wake on the fundamental mode for the three possible cases, namely, the pulse length is shorter than, longer than, or equal to that of the structure. Explicitly, this envelope

$$\frac{1}{M}\sum_{m=1}^{M}\left\{h\left[t - \frac{z}{c} - (m-1)T\right] - h\left[t - \frac{z}{c} - (m-1)T - \frac{d}{V}\right]\right\} \tag{3.3}$$

reveals the fact that the time it takes a microbunch to traverse the structure is d/V. Since $M \gg 1$ in all three cases, the *envelope* increases linearly until it reaches its maximum value. If the macrobunch is shorter than the structure, the peak value of the envelope is reached within the structure's domain and it remains at that value, until the first microbunch leaves the structure—at that time the envelope starts to decrease linearly. If the pulse length exactly equals that of the structure, the envelope has a triangular shape since as the last microbunch enters the structure the first one leaves. In the third case, when the macrobunch is longer than the structure, the peak value of the envelope is smaller than in the previous two cases, since a fraction of the microbunch is outside the structure.

In order to realize the difficulty associated with these profiles, one should bear in mind that either one of these envelopes propagates in the structure at $V \simeq c$:

$$E_W(\tau \equiv t - z/V) = 2\kappa^{(F)} \frac{q}{M} \sum_{m=1}^{M} \cos\left\{ \frac{2\pi c}{\lambda} \left[\tau - \tau_D - \frac{\lambda}{c}(m-1) \right] \right\} P\left[\tau - \tau_D - \frac{\lambda}{c}(m-1), \frac{d}{V} \right] \tag{3.4}$$

wherein the pulse function $P(t,T) = h(t) - h(t-T)$ represents an idealized envelope of the wake generated by a single microbunch. For comparison, any radiation pulse injected from outside propagates in the structure at $c\beta_{\rm gr}$. Such a pulse consists of two contributions: one that accelerates all the microbunches uniformly and the second designed to cancel the effect of the wake generated by the microbunches on their trailing counterparts—this last component will be referred to as the compensating field $[E_C(t,z)]$—or explicitly

$$G(t,z) = G_0 \cos\left[2\pi \frac{c}{\lambda} \left(t - \frac{z}{c} \right) \right] P\left(t - \frac{z}{V_{\rm gr}}, \tau_{\rm EM} \right) + E_C(t,z); \tag{3.5}$$

$\tau_{\rm EM}$ is defined in Eq. (3.2).

Our focus moves now toward determining the explicit functional form for this compensating field. For this purpose let us examine the variations in the kinetic energy lost by microbunches traversing the structure subject to their own wake [Eq. (3.4)] and ignoring transition radiation or reflections as well as the accelerating gradient. Bearing in mind that the trajectory of the microbunches may be approximated by

$$z_i(t) = c(t - \tau_D) - (i-1)\lambda \tag{3.6}$$

wherein $i = 1, 2, \ldots, M$, the energy gained by the ith microbunch is

$$\begin{aligned} U_i &\simeq \int_{\tau_D+(i-1)T}^{\tau_D+(i-1)T+d/c} dt c \frac{q}{M} \left\{ (-2q\kappa^{(F)}) \frac{1}{M} \times \sum_{m=1}^{M} h\left[t - \frac{z_i(t)}{c} - \tau_D - (m-1)\frac{\lambda}{c} \right] \right\} \\ &= -c\left(\frac{q}{M}\right)(2q\kappa^{(F)})\left(\frac{d}{c}\right)\left[\frac{1}{M} \sum_{m=1}^{M} h(i-m) \right] \\ &= -\left(\frac{q}{M}\right)(2q\kappa^{(F)} d)\left[\frac{1}{M}\left(i - 1 + \frac{1}{2} \right) \right]. \end{aligned} \tag{3.7}$$

This linear dependence of the kinetic energy gain on the location of the microbunch represented by the index i hints at the functional behavior of the compensating field. Including the overall accelerating field and imposing the condition that the overall U_i ought to be i independent, we found that the envelope of the accelerating field needs to vary linearly, i.e.,

$$G(t,z) \simeq \left[a + b \frac{1}{MT}\left(t - \frac{z}{V_{\rm gr}} \right) \right] \cos\left[2\pi \frac{c}{\lambda}\left(t - \frac{z}{c} \right) \right] \times P\left(t - \frac{z}{c\beta_{\rm gr}}, \tau_{EM} \right). \tag{3.8}$$

Explicitly, the amplitudes a and b are determined from the condition

$$\frac{q}{M} c \int_{\tau_D+(i-1)\lambda}^{\tau_D+(i-1)\lambda+d/c} \{G(t,z_i(t)) - E_w(t,z_i(t))\} = \frac{q}{M} d(G_0 - \kappa q) \tag{3.9}$$

where the right hand side was chosen such that the net kinetic energy gain of the macrobunch will be identical to the case in the previous subsection; consequently,

$$a = G_0 - \kappa q + \kappa^{(F)} q \left(\frac{1}{M} - \frac{1}{\beta_{\rm gr}} + 1 \right), \tag{3.10}$$

$$b = 2\kappa^{(F)} q.$$

This result reflects the fact that to overcome the difficulty associated with different propagating velocities of the two envelopes we have compromised on part of the constraints of the compensating field and injected an electromagnetic field. This has a phase velocity c but its *group velocity* is $c\beta_{\rm gr}$, and it has a trapezoidal shape with an *identical slope* to that of the wake. Moreover, because of the difference between the velocities of the electromagnetic pulse and macrobunch, an additional condition has been imposed, namely, the macrobunch must be *equal* to the length of the acceleration structure ($M\lambda = d$); otherwise, since the shape of the wake is trapezoidal, the gradient pulse cannot be "tailored" in a simple way such that all the microbunches gain the same amount of kinetic energy. Figure 5 illustrates the amplitude of the gradient at the *input* for three cases $V_{\rm gr} = 0.4c, 0.5c 0.6c$ and a loading factor $\alpha = \kappa^{(F)} q / G_0 \simeq 0.2$.

The electromagnetic energy injected into the system may be readily calculated based on Eq. (3.8) and, with it, the efficiency in this case is given by

$$\begin{aligned} \eta &\equiv \frac{\Delta U_{\rm kin}}{U_{\rm EM}} \\ &= \frac{[12(1-\beta_{\rm gr})\kappa_r] q(q_0 - q)}{3[q_0 - q + q\kappa_r(1/M + 1)]^2 + [q\kappa_r/\beta_{\rm gr}]^2} \\ &\simeq 12(1-\beta_{\rm gr})\beta_{\rm gr}^2 \frac{q(q_0 - q)}{q^2 + 3q_0^2\beta_{\rm gr}^2}. \end{aligned} \tag{3.11}$$

In the last step we assumed a large number of microbunches ($M \gg 1$) implying that most of the energy generated is in the fundamental ($\kappa_r \sim 1$). Subject to this condition the optimal charge is

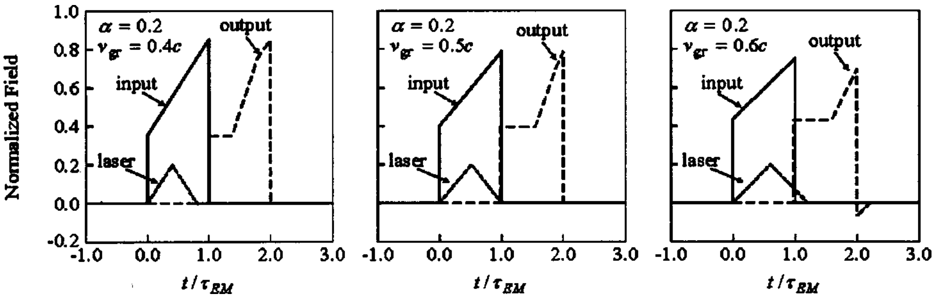

FIG. 5. Normalized field at the input and output of the acceleration structure for three values of the group velocity. Ignoring Ohmic and diffraction loss and then subtracting the adequate delay provides us with the information regarding the laser field as discussed in Sec. V.

$$q_{\rm opt} \simeq 3q_0\beta_{\rm gr}^2\left[\sqrt{1+\frac{1}{3\beta_{\rm gr}^2}}-1\right] \equiv q_0\xi \qquad (3.12)$$

and the maximum efficiency is

$$\eta_{\max} \simeq 12(1-\beta_{\rm gr})\beta_{\rm gr}^2\frac{\xi(1-\xi)}{\xi^2+3\beta_{\rm gr}^2}, \qquad (3.13)$$

both being illustrated in Fig. 6. Several facts are evident. (i) Maximum efficiency occurs for $q_{\rm opt} < q_0/2$. (ii) Maximum efficiency depends on the group velocity, reaching an upper limit of 45% when $\beta_{\rm gr} \simeq 0.3$. (iii) From the explicit expression for the optimal charge to be accelerated in order to get maximum efficiency ($q_{\rm opt} \simeq 0.3q_0$) we may mistakenly conclude that the charge to be accelerated is similar to the case described in Sec. II. This is not necessarily the case since $q_0 \equiv G_0/\kappa$ and for a given gradient the maximum amount of charge that can be accelerated is inversely proportional to the wake coefficient κ. But it was clearly revealed in Fig. 3 that this quantity may be reduced quite significantly by splitting the bunch into a train of microbunches—$\kappa^{(F)}$ remaining unchanged. Therefore, we should be able to enhance to some extent the efficiency and, in parallel, it is possible to increase the amount of charge accelerated.

IV. SINGLE BUNCH AND FEEDBACK

The last result is encouraging since it demonstrates that, by using a train of bunches accelerated by a tapered laser pulse, the maximum efficiency reaches the 50% level. To approach this efficiency level, the relative change of the amplitude is comparable to the efficiency, imposing a significant constraint on the optical system. In practice, the taper is expected to be moderate, implying low efficiency and low charge, and consequently it will be necessary to combine the "train of bunches" concept with the "feedback." However, before doing so let us examine the feedback concept decoupled from the train of microbunches.

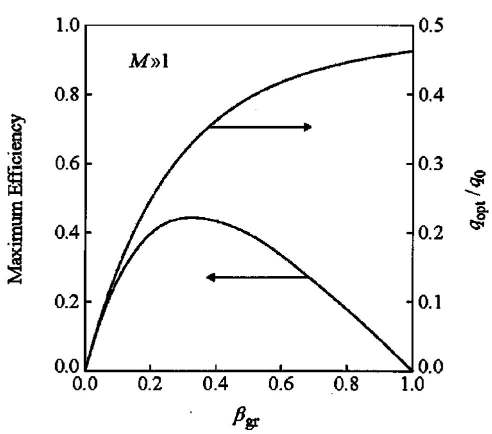

FIG. 6. The maximum efficiency [Eq. (3.13)] in the case of a macrobunch consisting of M microbunches. When this number is much larger than unity, the contribution of the high-frequency modes to the wake is negligible and as a result $\kappa(M)/\kappa^{(F)} \sim 1$. Our analysis indicates that in this case the peak efficiency is about 45%.

It was shown in Sec. II that the number of electrons that can be accelerated in a single macrobunch is of the order of 10^5; consequently, a repetition rate of about 1 GHz will be necessary for obtaining the desired flux of events at the interaction point (IP), namely, $\sim 10^{14}$ electrons/sec. It is therefore natural to build a feedback loop with an overall period which equals that of the macrobunches. Further support of this conclusion stems from examining the energy balance in the first subsection. It was systematically shown that only a small fraction (6%) of the electromagnetic energy is actually converted into kinetic energy; therefore, in principle, the remaining energy may be fed back into the system. In practice, it is necessary to add the energy for maintaining a constant gradient during each cycle as well as for global phase control.

A schematic of an acceleration module with a feedback loop is illustrated in Fig. 7. At its *input*, two contributions to the longitudinal electric field are assumed: that of an external laser pulse coupled into the acceleration structure (E^L) and that of the feedback loop ($E_{\rm FB}$).

At the *output* of the acceleration structure, the electric field is a superposition of two contributions: by virtue of Maxwell's equations, one is a linear function of the field at the input and the second is the projection of the wake on the fundamental (superscript F) mode $E_W = 2\kappa^{(F)}q$. Obviously,

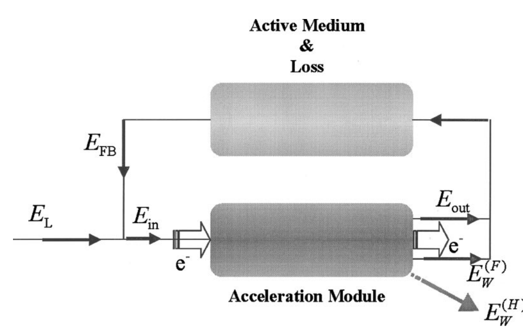

FIG. 7. Schematic of an acceleration structure that includes a feedback loop.

both waves occur at the output with a delay determined by the length of the structure and the group velocity; moreover, the coherent wake reduces the acceleration field. An additional contribution at the output is that of the high-frequency modes (superscript H) and it should be included in the overall energy balance since it has a significant effect on the efficiency of the structure.

Without significant loss of generality a *linear* feedback system is assumed to relate the input field ($E_{\rm FB}$) with that at the output of the acceleration structure. It is characterized by a delay ($\tau_{\rm FB}$) in the feedback section, by an overall gain $\bar{g}=ge^{-\psi}$ consisting of the gain (g) of the active medium and a loss parameter ($e^{-\psi}$); the latter being related to the quality factor of the system by $1-e^{-2\psi}\simeq 1/Q$. With these definitions we may determine the equation describing the field dynamics in terms of the output field

$$E_{\rm out}(t)=\bar{g}E_{\rm out}(t-T_{\rm rr})+E_L(t-d/c\beta_{\rm gr})-E_W^{(F)}(t-d/c\beta_{\rm gr}); \tag{4.1}$$

$T_{rr}\equiv\tau_{\rm FB}+d/c\beta_{\rm gr}$ being the periodicity of the macrobunch (inversely proportional to the repetition rate). For more details see Appendix C. Once this equation has been established our goal is to determine the corresponding efficiency of the system.

With this purpose in mind it is convenient to revert to the pulse function $P(t,d/V)$ defined in the context of Eq. (3.4) representing an *idealized* envelope of the wake generated by a single micro bunch moving with a velocity $V\simeq c$ and injected in a structure. This pulse shape may be understood in terms of the power generated by this microbunch: the latter starts generating Čerenkov radiation as it enters the structure ($t=0$) and it ceases the emission as it leaves ($t=d/V$)—transition radiation and reflections are ignored. In addition, we disregard the higher eigenmodes excited in the system. They decay on a time scale that is determined by the overall quality factor of the system—in fact, it may be assumed that this field is filtered out by the feedback system. In terms of the pulse function introduced above, a typical solution has the form of an "infinite" series of pulses with periodicity T_{rr}, e.g.,

$$E_{\rm out}(t)=\sum_{n=0}^{\infty}E_n^{(\rm out)}P(t-nT_{rr}). \tag{4.2}$$

In what follows we shall focus on one cycle, after all transients associated with turning on the system have decayed. A similar approach for evaluation of the efficiency as in Sec. II is adopted. Clearly, the kinetic energy is the same, but the injected energy has two contributions: the *external* laser field

$$U_{\rm laser}=\frac{\lambda^2E_L^2}{Z_{\rm int}}\tau_{\rm EM} \tag{4.3}$$

and that of the *active medium*

$$U_{\rm active}=(g^2-1)U_{\rm out}=\frac{\lambda^2(g^2-1)(G_0-2\kappa^{(F)}q)^2}{Z_{\rm int}}\tau_{\rm EM} \tag{4.4}$$

In equilibrium by virtue of energy conservation, these two quantities balance the energy that leaves the system: $U_{\rm loss}$ and $\Delta U_{\rm kin}$; $U_{\rm loss}$ is the total energy "wasted" in the loop, which is inversely proportional to the quality factor mentioned above, and proportional to the total energy in the loop represented here by the energy at the output of the loop, i.e., $U_{\rm loss}\simeq U_{\rm out}/Q$. With these observations the efficiency is defined by

$$\eta=\frac{\Delta U_{\rm kin}}{U_{\rm laser}+U_{\rm active}}=\frac{\Delta U_{\rm kin}}{\Delta U_{\rm kin}+U_{\rm loss}}=\frac{1}{1+U_{\rm out}/Q\Delta U_{\rm kin}}. \tag{4.5}$$

Substituting adequate definitions including Eqs. (2.5) and (2.6), we obtain

$$\eta=\frac{1}{1+\dfrac{1}{Q}\dfrac{1}{\kappa_r4q(q_0-q)/(q_0-2\kappa_rq)^2}}. \tag{4.6}$$

In the case of a single bunch the relative wake coefficient $\kappa_r=\kappa^{(F)}/\kappa\ll 1$ is much smaller than unity; therefore, the efficiency may be simplified to read

$$\eta\simeq\frac{1}{1+\dfrac{1}{Q}\dfrac{1}{\kappa_r4q(q_0-q)/q_0^2}}. \tag{4.7}$$

It may be readily shown that the maximum efficiency occurs for $q_{\rm opt}=q_0/2$ and is given by

$$\eta_{\max}\simeq\frac{1}{1+1/Q\kappa_r}\simeq\begin{cases}1, & Q\to\infty,\\ \kappa_r, & Q\to 1,\end{cases} \tag{4.8}$$

revealing that the efficiency approaches the 100% limit if the quality factor is sufficiently large, and it equals the efficiency in Eq. (2.9) corresponding to lack of feedback. Two important comments are in order at this stage. First, the number of electrons that can be accelerated remains as in Sec. II—which is rather low. Although some tradeoff is possible by reducing the efficiency at the expense of increasing the total amount of charge, still the latter is limited to $q_0=G_0/\kappa$, which in the case of one microbunch is minuscule. Second, it has been tacitly assumed that the system has reached steady state, implying that in any given cycle the energy from the external laser and from the active medium exactly compensate the energy lost to Ohmic processes or diffraction and energy transferred to electrons.

V. MULTIPLE MICROBUNCHES AND FEEDBACK

Combining multiple microbunches and a feedback loop will enable us to enhance both the efficiency as well as the number of electrons to be accelerated. Subject to the condition $d \simeq M\lambda$ and a trapezoidal pulse (see Fig. 5) propagating at the group velocity, then the gradient as experienced by a test charge is

$$E_{\rm in}(t,z) = \left(a + b\frac{t - z/V_{\rm gr}}{TM}\right)\cos\left[\omega_0\left(t - \frac{z}{c}\right)\right]P\left(t - \frac{z}{V_{\rm gr}}, \tau_{\rm EM}\right) \tag{5.1}$$

which is a homogeneous solution of the wave equation, contrary to the wake of the macrobunch, which is a nonhomogeneous solution,

$$E_w(t,z) = -2\kappa^{(F)}q\frac{1}{M}\sum_{m=1}^{M} P\left[t - \tau_D - \frac{z}{c} - \frac{\lambda}{c}(m-1), \frac{d}{c}\right] \times\cos\left[\omega_0\left(t - \frac{z}{c}\right)\right]; \tag{5.2}$$

the amplitudes a and b have been defined in Eq. (3.10).

As already indicated in Sec. III, with these field components all M microbunches are equally accelerated and the kinetic energy of the macrobunch is $\Delta U_{\rm kin}=q(G_0-\kappa q)d$. However, in contrast to the case when there is no feedback, the form of the output signal becomes critical since it must be self-consistent with the signal from the feedback loop and the gain medium. Figure 5 illustrates this output signal for three group velocities $V_{\rm gr}=0.4c, 0.5c, 0.6c$ and a loading factor$\alpha=\kappa^{(F)}q/G_0 \simeq 0.2$. It is evident that the front of the pulse at the output is constant for the time duration that electrons traverse the interaction region. Beyond that period, the output varies according to the injected input signal and the wake of the microbunch.

Since it has been tacitly assumed that the active section exactly compensates any radiation loss, subtracting the output from the input (with adequate delay adjustment) provides us with the exact shape of the *external* laser field necessary to compensate the beam loading, i.e., the wake projection on the fundamental mode. In all three cases illustrated in Fig. 5, the external laser pulse has a triangular shape since the wake itself has this form—after imposing the condition $d=M\lambda$. The time scale is normalized to the electromagnetic pulse duration ($\tau_{\rm EM}=d/c\beta_{\rm gr}$) and the length of the external laser pulse is identical with that of the electrons $\tau_{\rm laser} \simeq 2d/c$. It therefore becomes evident that as $V_{\rm gr}=0.4c$ its normalized length is shorter than that of the electromagnetic pulse, if $V_{\rm gr}=0.5c$ it exactly equals it, and for the case when $V_{\rm gr}=0.6c$ the wake duration (and thus the external laser pulse) is longer than the accelerating electromagnetic field.

To maintain this self-consistent solution two equally important contributions are strictly necessary: first, the active medium needs to exactly compensate all radiation loss and, second, the external laser field needs to compensate the beam loading. In principle, it is possible that *part* of the energy lost to high-order modes or optical components will be compensated by the external laser. This is easily accomplished if the group velocity is designed to be exactly $V_{\rm gr}=0.5c$ since both the external laser pulse and the accelerating electromagnetic field have the same duration; otherwise, the form of the laser pulse is difficult to implement since it is no longer triangular. However, from the perspective of energy balance, it does not matter what fraction of the energy is injected via the active medium or the external field—in this context, we ignore noise phenomena associated with spontaneous radiation in the active medium or finite momentum spread of the electrons.

Evaluation of the efficiency requires establishing the total energy fed into the system during one cycle. Based on Fig. 5 we realize that the energy linked to the *external laser* field is

$$U_{\rm laser} = \frac{\lambda^2}{Z_{\rm int}}2\int_0^{d/c} dt\left[(\kappa^{(F)}q)\frac{ct}{d}\right]^2 = \frac{2}{3}\frac{\lambda^2}{Z_{\rm int}}(\kappa^{(F)}q)^2\frac{d}{c}; \tag{5.3}$$

thus our next step for the evaluation of the efficiency is to calculate the energy provided by the *active medium*. At the input of the active section the energy is assumed to equal the energy at the output of the acceleration structure, $U_{\rm out}$; therefore, denoting by g the gain of the active section at the frequency of interest we get

$$U_{\rm active} = (g^2-1)U_{\rm out}. \tag{5.4}$$

Further assuming that the system has reached steady state, these two energy contributions "injected" into the system compensate for the energy that leaves the system, namely, the net kinetic energy and energy wasted via Ohmic loss as well as diffraction. These two last contributions are proportional to the energy stored in the entire loop represented by the energy at the output of the acceleration structure, $U_{\rm out}$, and inversely proportional to the effective quality factor Q of the entire loop, or explicitly $U_{\rm loss}=U_{\rm out}/Q$. By virtue of energy conservation $U_{\rm laser}+U_{\rm active}=\Delta U_{\rm kin}+U_{\rm loss}$; therefore we may define the system's efficiency as

$$\eta \equiv \frac{\Delta U_{\rm kin}}{U_{\rm laser}+U_{\rm active}} = \frac{\Delta U_{\rm kin}}{\Delta U_{\rm kin}+U_{\rm loss}} = \frac{1}{1+U_{\rm out}/Q\Delta U_{\rm kin}}. \tag{5.5}$$

Based on the plot in Fig. 5 it can be shown that

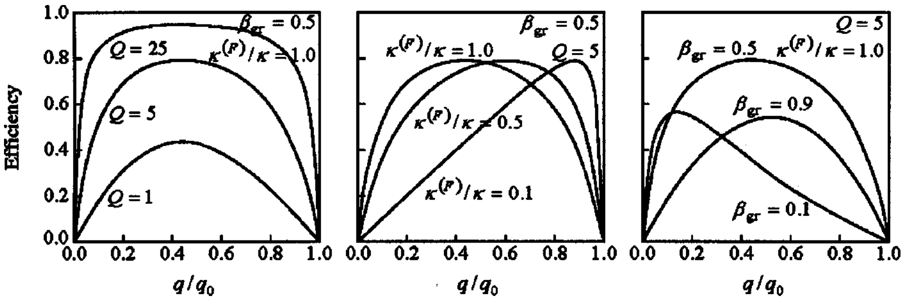

FIG. 8. Efficiency as a function of the accelerated charge with the quality factor, wake parameter, and group velocity as parameters.

$$U_{\text{laser}} = \frac{\lambda^2}{Z_{\text{int}}} \frac{d}{c} \frac{b^2}{6},$$

$$U_{\text{out}} = \frac{\lambda^2}{Z_{\text{int}}} \frac{d}{c} \times \begin{cases} a^2 \dfrac{1}{\beta_{\text{gr}}} + ab\left(\dfrac{1}{\beta_{\text{gr}}^2} - 2\right) + b^2 \dfrac{1}{3}\left(\dfrac{1}{\beta_{\text{gr}}^3} - 4\right), & \beta_{\text{gr}} \leqslant 0.5 \\ a^2 \dfrac{1}{\beta_{\text{gr}}} + 2ab\left(\dfrac{1}{\beta_{\text{gr}}} - 1\right)^2 + b^2\left(\dfrac{1}{\beta_{\text{gr}}^3} - \dfrac{2}{\beta_{\text{gr}}^2} + \dfrac{4}{3}\right), & \beta_{\text{gr}} \geqslant 0.5 \end{cases} \tag{5.6}$$

leading to the following explicit expression for the efficiency:

$$\eta = \begin{cases} \dfrac{1}{1 + \dfrac{1}{Q}\dfrac{q}{q_0 - q}\dfrac{\beta_{\text{gr}}}{1-\beta_{\text{gr}}}\dfrac{\kappa^{(F)}}{\kappa}\left[\left(\dfrac{a}{b}\right)^2 \dfrac{1}{\beta_{\text{gr}}} + \dfrac{a}{b}\left(\dfrac{1}{\beta_{\text{gr}}^2} - 2\right) + \dfrac{1}{3}\left(\dfrac{1}{\beta_{\text{gr}}^3} - 4\right)\right]}, & \beta_{\text{gr}} \leqslant 0.5 \\ \dfrac{1}{1 + \dfrac{1}{Q}\dfrac{q}{q_0 - q}\dfrac{\beta_{\text{gr}}}{1-\beta_{\text{gr}}}\dfrac{\kappa^{(F)}}{\kappa}\left[\left(\dfrac{a}{b}\right)^2 \dfrac{1}{\beta_{\text{gr}}} + 2\left(\dfrac{a}{b}\right)\left(\dfrac{1}{\beta_{\text{gr}}} - 1\right)^2 + \left(\dfrac{1}{\beta_{\text{gr}}^3} - \dfrac{2}{\beta_{\text{gr}}^2} + \dfrac{4}{3}\right)\right]}, & \beta_{\text{gr}} \geqslant 0.5 \end{cases} \tag{5.7}$$

According to this result the efficiency depends on four parameters: normalized charge (q/q_0), group velocity (β_{gr}), relative wake coefficient ($\kappa_r = \kappa^{(F)}/\kappa$), and quality factor ($Q$). Figures 8 and 9 illustrate the main features of the efficiency and its dependence on these parameters.

The left frame in Fig. 8 shows that larger quality factors lead to higher efficiency and reduced sensitivity to variations around the peak-efficiency. Even for a low quality factor the maximum efficiency may reach the 40% level since the macrobunch is formed by a large number of microbunches—as discussed in Sec. III. The direct impact of the relative wake coefficient ($\kappa_r = \kappa^{(F)}/\kappa$) is illustrated in the middle frame where it is assumed that the latter is not dependent on the group velocity. We see rough indications that the maximum efficiency is independent of κ_r but its off-peak behavior strongly depends on this parameter. Similar to the left frame, as the wake parameter approaches unity, the sensitivity to variations in the number of electrons accelerated diminishes; the same holds for the case when the group velocity is one-half c—see right frame.

Further support for these findings is revealed by Fig. 9. Its left frame shows the maximum efficiency and the optimal charge where it occurs, as a function of the group velocity. For a large quality factor there are negligible variations in the efficiency when the group velocity changes ($0.2 \leqslant \beta_{\text{gr}} \leqslant 0.8$). For all three cases ($Q = 1, 5, 25$) the optimal charge to be accelerated is independent of the quality factor but it varies significantly with the group velocity. The opposite holds

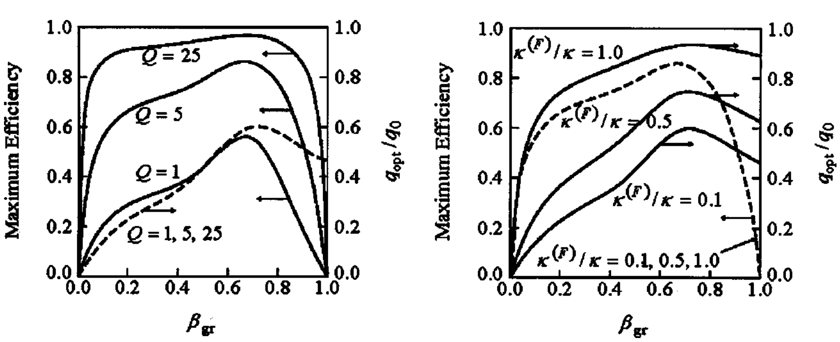

FIG. 9. Maximum efficiency and the optimal charge where it occurs as a function of the group velocity with the wake parameter and the quality factor as parameters.

TABLE I. Main estimates of the study.

Single bunch	
Without feedback	With feedback
$\eta=\kappa_r\dfrac{4q(q-q_0)}{q_0^2}$	$\eta\simeq\dfrac{1}{1+\dfrac{1}{Q}\dfrac{1}{\kappa_r 4q(q-q_0)/q_0^2}}$
$\eta_{\max}=\kappa_r$	$\eta_{\max}\simeq\dfrac{1}{1+1/Q\kappa_r}$
$q_{\mathrm{opt}}=\dfrac{1}{2}q_0$	$q_{\mathrm{opt}}=\dfrac{1}{2}q_0$
Multiple bunches	
Without feedback	With feedback
$\eta\simeq\dfrac{12(1-\beta_{\mathrm{gr}})\kappa_r[q(q_0-q)]}{3[q_0-q+q\kappa_r]^2+[q\kappa_r/\beta_{\mathrm{gr}}]^2}$	$\eta=\dfrac{1}{1+\dfrac{1}{Q}\dfrac{q}{q_0-q}\dfrac{\beta_{\mathrm{gr}}}{1-\beta_{\mathrm{gr}}}\kappa_r F(q,\beta_{\mathrm{gr}},\kappa_r,q_0)}$
$\eta_{\max}\simeq 12(1-\beta_{\mathrm{gr}})\beta_{\mathrm{gr}}^2\dfrac{\xi(1-\xi)}{\xi^2+3\beta_{\mathrm{gr}}^2}$; $\kappa_r\simeq 1$	$F(q,\beta_{\mathrm{gr}},\kappa_r,q_0)=\begin{cases}\left(\dfrac{a}{b}\right)^2\dfrac{1}{\beta_{\mathrm{gr}}}+\dfrac{a}{b}\left(\dfrac{1}{\beta_{\mathrm{gr}}^2}-2\right)+\dfrac{1}{3}\left(\dfrac{1}{\beta_{\mathrm{gr}}^3}-4\right) & \beta_{\mathrm{gr}}\leqslant 0.5\\ \left(\dfrac{a}{b}\right)^2\dfrac{1}{\beta_{\mathrm{gr}}}+2\left(\dfrac{a}{b}\right)\left(\dfrac{1}{\beta_{\mathrm{gr}}}-1\right)^2+\left(\dfrac{1}{\beta_{\mathrm{gr}}^2}-\dfrac{2}{\beta_{\mathrm{gr}}^2}+\dfrac{4}{3}\right) & \beta_{\mathrm{gr}}\geqslant 0.5\end{cases}$
$q_{\mathrm{opt}}\simeq\xi q_0$	
$\xi\simeq 3\beta_{\mathrm{gr}}^2\left[\sqrt{1+\dfrac{1}{3\beta_{\mathrm{gr}}^2}}-1\right]$	$\dfrac{a}{b}=\dfrac{\kappa_r}{2q}\left[q_0-q-q\kappa_r\left(\dfrac{1}{\beta_{\mathrm{gr}}}-1\right)\right]$

when the quality factor is kept constant $(Q=5)$ and the normalized wake parameter is taken to be $\kappa_r=0.1, 0.5, 1.0$. Clearly, the maximum efficiency is independent of the latter but the optimal charge to be accelerated varies significantly as a function of the group velocity and the wake parameter. It is important to point out that these results have to be considered only as a general trend since the assumption that the group velocity and the wake parameter are independent is a very stringent constraint. For a particular structure their interrelation should be accounted for and, as a result, the exact character of these plots may vary from one structure to another. Nevertheless, it has been checked based on a simple dielectric loaded structure that the general trend of an efficiency approaching the 90% level can be validated assuming a quality factor of 25 and a group velocity of $0.5c$. Moreover, compared to the case of a single bunch and no feedback, the amount of charge can be up to ten times higher since the wake parameter κ can be significantly smaller in the case of a large train of bunches—see Fig. 3.

VI. DISCUSSION

Using simplified models we have demonstrated that the efficiency of a future optical collider may be enhanced from a few percent, in the simplest configuration when a single bunch is accelerated and the accelerating pulse is eventually dumped, to over 90% when feedback and multiple microbunch schemes are employed. Table I summarizes the main estimates of this study referring to four main parameters: efficiency, optimal charge where maximum efficiency occurs, group velocity, and relative wake coefficient $\kappa_r=\kappa^{(F)}/\kappa$. It is shown that the last determines the efficiency in the absence of feedback and when only a single microbunch is accelerated—see Eq. (2.9). In this expression, κ is the wake coefficient which, given the charge of the bunch, determines the overall decelerating field. In principle, this field has an infinite spectrum but of special interest is the projection of this field on the fundamental mode—the one responsible for the acceleration. The corresponding coefficient is denoted by $\kappa^{(F)}$ and the typical efficiency is 6%.

By splitting the same amount of charge into a train of microbunches with a periodicity identical to that of the fundamental mode, it is possible to substantially enhance the efficiency (45%) since the high-frequency content of the wake is dramatically reduced by the "microstructure" of the bunch. In parallel, the charge for which maximum efficiency occurs is inversely proportional to the wake coefficient, $q_{\rm opt} \propto 1/\kappa(M)$; therefore the amount of accelerated electrons is higher. A direct by-product of splitting the bunch into a train of microbunches is different loading along the macrobunch. For this reason, the accelerating gradient ought to be *tapered* in order to compensate this space dependence of the loading; otherwise, the kinetic energy gain along the bunch will not be uniform. Another process that was taken into consideration is related to the different propagation velocities of the bunch and the wave in the structure. Specifically, since the wake propagates virtually at the speed of light whereas the gradient's envelope propagated at $c\beta_{\rm gr}$, it was convenient to impose a constraint on the pulse duration, namely, the macrobunch must be *equal* to the length of the acceleration structure $M\lambda = d$.

Recycling part of the electromagnetic energy leads to much higher efficiencies ($\sim$90%) in either of the cases: single bunch or multiple bunches. However, while in the case of a single bunch the optimal amount of charge that can be accelerated is virtually identical to that in the absence of feedback, in the case of a train of microbunches, the overall charge can be significantly larger since the wake coefficient is reduced $(q_0 \propto 1/\kappa)$. To a large extent, the efficiency is determined by the quality factor (Q) of the system; however, to maintain this self-consistent solution two equally important contributions are strictly necessary: first, the active medium needs to exactly compensate all radiation loss and, second, the external laser field needs to compensate for the beam loading. As illustrated in Fig. 5, this is easily accomplished if the group velocity is designed to be exactly $V_{\rm gr} = 0.5c$ since both the external laser pulse and the accelerating electromagnetic field have the same duration, implying a triangular shape for the external laser pulse.

Although the present analysis makes a clear distinction between the energy provided by the active medium (amplifier) and that supplied by the external laser, in practice, it may be possible to provide all the energy by the external laser. We believe that by incorporating the active medium in the feedback loop we effectively produce a high-Q "cavity" similar to a superconducting one; therefore, the analogy to well-known systems becomes clearer. Moreover, the external laser has to generate a pulse that is shaped to the requirements of the "eigenmode" of the system, namely, the feedback loop and electron pulse. On the other hand, in the amplifier, no such shaping is necessary, reducing significantly the number of passive optic components involved and consequently diminishing the overall radiation loss.

Contrary to the amplifier section whose energy contribution may be supplied by the external laser, the latter is irreplaceable since beyond providing the necessary energy to compensate for the beam loading effect it locks the phase of the field in each acceleration module at the desired value. This observation brings us to an additional tacit assumption that needs to be reiterated. Our reference for the applied energy was the energy provided by the active medium and external laser. In either case we took the wall plug to light efficiency as ideal (100%), which, as already indicated, is far from being the case; however, we used that as our reference level in this study. Any efficiency mentioned above, either the one corresponding to the simple configuration in Sec. II or the more complex one in Sec. V, ought to be multiplied by the wall plug to light efficiency of the laser system regardless if it operates as a source (external laser) or amplifier (active medium).

Before we conclude it is important to address three more topics: the first is the system's stability, the second is the feasibility of pulse tapering as required in the configurations analyzed in Secs. III and V, and the third is accumulated noise. For analyzing the *stability* of the system let us assume a slight deviation in the energy stored in the loop δU. In Secs. IV and V we used as reference for this quantity the energy at the output of the acceleration structure, $U_{\rm out}$, therefore let us assume

$$U_{\rm out} \to U_{\rm out} + \delta U \tag{6.1}$$

In zero order, during one cycle of the system (T_{rr}) this change does not affect the net kinetic energy gain or the external laser pulse. As a result, this small variation is anticipated to "dissipate" according to

$$\begin{aligned}\frac{d}{dt}\delta U &= \frac{1}{T_{rr}}(U_{\rm laser} + U_{\rm active} - \Delta U_{\rm kin} - U_{\rm loss})\\ &= \frac{1}{T_{rr}}\bigg[U_{\rm laser} + (g^2-1)(U_{\rm out}+\delta U) - \Delta U_{\rm kin}\\ &\quad - \frac{1}{Q}(U_{\rm out}+\delta U)\bigg]\\ &= \frac{1}{T_{rr}}\left[(g^2-1)\delta U - \frac{1}{Q}\delta U\right]\\ &= \frac{1}{T_{rr}}\left[g^2-1-\frac{1}{Q}\right]\delta U,\end{aligned} \tag{6.2}$$

where in the last step energy conservation, $U_{\rm laser}+(g^2-1)U_{\rm out} = \Delta U_{\rm kin} + U_{\rm out}Q$, was assumed. Consequently, to ensure stability, these fluctuation ought to decay during several cycles; therefore, it is necessary to impose

$$g^2 - 1 < \frac{1}{Q}. \tag{6.3}$$

Now a few comments on our tacit assumption regarding *pulse shaping* of a picosecond long laser pulse as reflected in Eq. (3.8). There is an extensive research effort in the world with the sole purpose of shaping pico- or femtosecond laser pulses [17]. A variety of methods are being used: starting from fixed masks, through adaptive pulse shapes controlled by a computer, including liquid crystal devices, acousto-optic modulators, and ending with movable or deformable mirrors. A short list of applications of picosecond and femtosecond preshaped laser pulses is varied, including dark soliton experiments in nonlinear optics [18] or ultrashort pulse communications based on spectral phase encoding and decoding, where different users share a common fiber optic medium based on the use of different spectral codes (code division multiple access). In addition, it is possible to preshape a laser pulse to compensate for dispersion along an optical fiber [19]. The progress in this area indicates that the techniques have matured during the past decade and there is no reason to refrain from employing them within the framework of a future optical collider.

Finally, on the issue of noise and instabilities in a system with feedback: It is anticipated that the noise has three main sources, the randomness associated with the distribution of electrons, the external laser, and the active medium itself since it generates spontaneous radiation. Conceptually, beam break up (BBU) is not expected to be significantly different in an optical structure than in a regular microwave accelerator. On the one hand, in our favor is the fact that both photonic band-gap structures and symmetric Bragg fibers confine fewer modes due to their geometric characteristics as well as the frequency dependence of dielectrics in this frequency range. Consequently, better control is anticipated on the suppression of high-order modes. On the other hand, the radius of the electron beam, relative to the operating wavelength, is expected to be significantly larger comparing to this ratio in a microwave accelerator—and this may lead to enhanced sensitivity to BBU. The active medium as well as the external laser contribute their share to the noise that accumulates in the ring. Like the efficiency, their relative contribution will be determined to a large extent by the quality factor, and the latter's choice will eventually be a tradeoff between high efficiency, stability [Eq. (6.3)], and acceptable noise level that we may allow to develop in the loop.

ACKNOWLEDGMENTS

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APPENDIX A

The purpose of this appendix is to demonstrate that the condition for full overlap between the point charge and the radiation pulse leads to Eq. (2.6). In the acceleration structure the radiation pulse propagates at a velocity $c\beta_{\mathrm{gr}}$ whereas the electrons propagate virtually at c. As the point charge enters the acceleration structure, it encounters the field at the back end of the radiation pulse—see left frame of Fig. 10

As the point charge enters the structures there are two possibilities. On the one hand, if the radiation pulse is too short, it may pass the front end of the radiation pulse which obviously is undesirable since the point charge is not accelerated during all the time it spends in the structure. On the other hand, if the radiation pulse is too long, there will be a fraction of this pulse which will have no contribution to the acceleration process in the structure. Again, this is undesirable since energy is wasted and the overall efficiency drops. For these reasons, the pulse duration is taken such that the

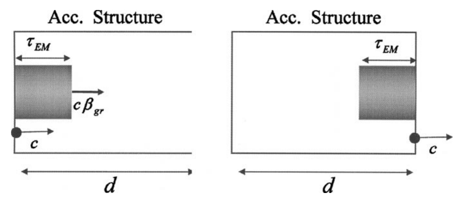

FIG. 10. As the point charge traverses the acceleration structure, it must experience an electromagnetic field throughout the time it spends in the structure.

point charge exactly overlaps the pulse in the acceleration structure. Explicitly, it takes the point charge a time d/c to traverse the structure. In a similar way, the length of the pulse in the structure is given by its duration and its propagation velocity $\tau_{\mathrm{EM}}c\beta_{\mathrm{gr}}$. With these observations, we realize that the time it takes the front end of the radiation pulse to reach the output end of the structure is given by $(d-\tau_{\mathrm{EM}}c\beta_{\mathrm{gr}})/c\beta_{\mathrm{gr}}$. Clearly, the overlap constraint imposed earlier implies that this time ought to equal it takes the point charge to traverse the structure, namely,

$$\frac{d}{c}=\frac{d-\tau_{\mathrm{EM}}c\beta_{\mathrm{gr}}}{c\beta_{\mathrm{gr}}}\Rightarrow\tau_{\mathrm{EM}}=\frac{d}{c}\left(\frac{1}{\beta_{\mathrm{gr}}}-1\right),$$

which is exactly Eq. (2.6).

The delay time (τ_D) between the front of the radiation pulse and point charge must equal the duration of the pulse subject to the assumption that the group velocity in the space in front of the acceleration structure is c, namely, $\tau_D=\tau_{\mathrm{EM}}$ (see Fig. 11).

APPENDIX B

In this appendix our goal is to demonstrate the power spectrum dependence on the number of microbunches, as-

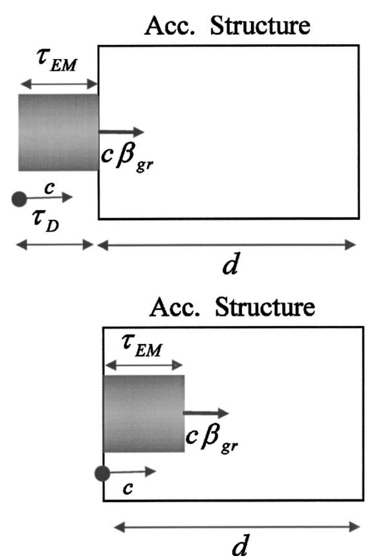

FIG. 11. Outside the acceleration structure there is a delay (τ_D) between the point charge and the front end of the radiation pulse. This delay exactly equals the pulse duration assuming that the propagation is in vacuum $(\beta_{\mathrm{gr}}=1)$.

suming that the overall charge in the macrobunch is maintained constant. The structure is assumed to be closed and therefore the spectrum is discrete. Moreover, the structure is designed such that the phase velocity of the first mode is c at the operating wavelength (λ_0) and, correspondingly, the latter is also the separation of any two microbunches.

Our starting point is the wake in a closed structure as presented in Eq. (2.3), which for convenience is repeated next:

$$E_z(r=0,\tau\equiv t-z/c)=q\kappa\sum_{n=1}^{\infty}W_n\cos(\omega_n\tau)[2h(\tau)].$$

Let us label by ν each microbunch such that $\nu=1,2,\ldots,M$. Since the total emitted power is given by $P=\int dv J_z E_z$ we can explicitly find that this power is given by

$$P=\kappa q^2 v\sum_{n=1}^{\infty}W_n 2\left\langle\left\langle\cos\left[\frac{\omega_n}{\omega_1}2\pi(\mu-\nu)\right]h[2\pi(\mu-\nu)]\right\rangle_{\mu}\right\rangle_{\nu}$$

wherein $\langle\cdots\rangle_\nu\equiv(1/M)\sum_{\nu=1}^{M}\cdots$. Using geometric series expansion and taking into account the value of the step function at zero, the double average may be shown to equal

$$\left\langle\left\langle\cos\left[\frac{\omega_n}{\omega_1}2\pi(\mu-\nu)\right]h[2\pi(\mu-\nu)]\right\rangle_{\mu}\right\rangle_{\nu}=\frac{1}{2}\frac{\mathrm{sinc}^2[\pi(\omega_n/\omega_1)M]}{\mathrm{sinc}^2[\pi\omega_n/\omega_1]}$$

and consequently

$$P(M)=\kappa q^2 v\sum_{n=1}^{\infty}W_n\frac{\mathrm{sinc}^2[\pi(\omega_n/\omega_1)M]}{\mathrm{sinc}^2[\pi\omega_n/\omega_1]}$$

which for a relativistic particle is exactly the expression in Eq. (3.1); here $\mathrm{sinc}(x)\equiv\sin(x)/x$.

APPENDIX C

Here we bring a detailed account of the arguments which led to Eq. (4.1). At the output of the acceleration structure, the electric field is a superposition of two contributions: by virtue of Maxwell's equations, one is a linear function of the field at the input and the second is the projection of the wake on the fundamental (superscript F) mode $E_W=2\kappa^{(F)}q$

$$E_{\mathrm{out}}(t)=\int_{-\infty}^{\infty}dt' L(t|t')[E_{\mathrm{in}}(t')-E_W(t')];$$

note that it was explicitly assumed that the coherent wake reduces the acceleration field. In a similar way we assume a linear operator relating the field at the input with that at the output of the accelerating structure, and in addition there is the external laser field (E_L) injected into the system,

$$E_{\mathrm{in}}(t)=E_L(t)+\int_{-\infty}^{\infty}dt' G(t|t')E_{\mathrm{out}}(t').$$

Without significant loss of generality, the linear operator representing the propagation through the traveling wave structure (ignoring dispersion or radiation loss of any kind) is given by $L(t|t')\simeq\delta(t'-t+d/c\beta_{\mathrm{gr}})$; the last term $(d/c\beta_{\mathrm{gr}})$ represents the delay of the envelope of the pulse. In a similar way, the operator representing the feedback loop includes the feedback delay (τ_{FB}), the gain of the active medium (g), and the overall loss in the entire loop $(e^{-\psi})$; thus $G(t|t')\simeq ge^{-\psi}\delta(t'-t+\tau_{\mathrm{FB}})$. Consequently, the difference equation for the amplitude at the output is

$$E_{\mathrm{out}}(t)=\bar{g}E_{\mathrm{out}}(t-T_{rr})+E_L(t-d/c\beta_{\mathrm{gr}})-E_W(t-d/c\beta_{\mathrm{gr}});$$

here the overall gain is denoted by $\bar{g}=ge^{-\psi}$ and $T_{\mathrm{rr}}\simeq\tau_{\mathrm{FB}}+d/c\beta_{\mathrm{gr}}$ is the periodicity of the macrobunch (inversely proportional to the repetition rate).

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